PRACTICE & PROBLEM SOLVING

UNDERSTAND

10. Construct Arguments Tamara says that raising the number i to any integer power results in either -1 or 1 as the result, since $i^2 = -1$. Do you agree with Tamara? Explain.

11. Error Analysis Describe and correct the error a student made when dividing complex numbers.
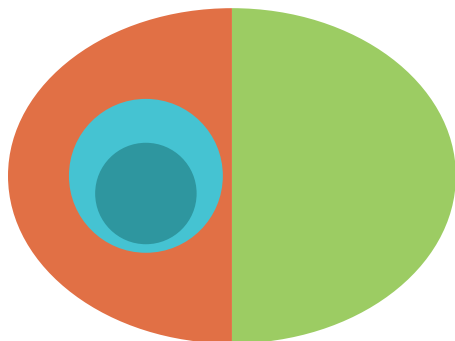
$$\frac{1+i}{3-i} =$$

$$\frac{1+i}{3-i} \cdot \frac{1-i}{3+i} =$$

$$\frac{1-i^2}{9-i^2} =$$

$$\frac{2}{10} \quad \textcolor{red}{\times}$$

12. Higher Order Thinking Label the diagram with the following sets of numbers:

1. complex numbers   **4.** integers

2. real numbers   **5.** rational numbers

3. imaginary numbers

Include an example of each type of number in the diagram.

13. Generalize Write an explicit formula, in standard form, to find the quotient of two complex numbers. Use the numbers $a + bi$ and $c + di$.

PRACTICE

Use square roots to solve each equation over the complex numbers. SEE EXAMPLE 1

14. $x^2 = -5$ **15.** $x^2 = -0.01$

16. $x^2 = -18$ **17.** $x^2 = (-1)^2$

Add or subtract. Write the answer in the form $a + bi$. SEE EXAMPLE 2

18. $(3 - 2i) - (-9 + i)$ **19.** $(5 + 1.2i) + (-6 + 0.8i)$

20. $(2i) - (2i - 11)$ **21.** $13 + 2i - 4 - 8i$

22. $\frac{3-i}{4} - \frac{2+i}{3}$ **23.** $4.5i - 4.5 + 3.5i + 2.5$

Write each product in the form $a + bi$. SEE EXAMPLE 3

24. $(11i)(3i)$ **25.** $(3i)(5 - 4i)$

26. $(5 - 2i)(5 + 2i)$ **27.** $(8 + 3i)(8 + 3i)$

28. $\frac{1}{3}i(3 + 6i)$ **29.** $(-2i + 7)(7 + 2i)$

Write each quotient in the form $a + bi$.
SEE EXAMPLE 4

30. $\frac{12}{1-i}$ **31.** $\frac{5}{6+2i}$

32. $\frac{6+12i}{3i}$ **33.** $\frac{4-4i}{1+3i}$

Factor the sums of two squares. SEE EXAMPLE 5

34. $4x^2 + 49$ **35.** $x^2 + 1$

36. $36 + 100a^2$ **37.** $18y^2 + 8$

38. $\frac{1}{4}b^2 + 25$ **39.** $x^2 + y^2$

Solve each equation. SEE EXAMPLE 6

40. $x^2 + 81 = 0$ **41.** $25x^2 + 9 = 0$

42. $x^2 = -16$ **43.** $4 + 49y^2 = 0$

44. $y^2 + 1 = 0$ **45.** $x^2 + \frac{1}{4} = 0$

APPLY

46. Model With Mathematics The two resistors shown in the circuit are referred to as *in series*. The total resistance of the resistors is given by the formula $R_T = \frac{1}{R_1} + \frac{1}{R_2}$.

$R_1 = 4 + 2i$ ohms
$R_2 = 1 + i$ ohms
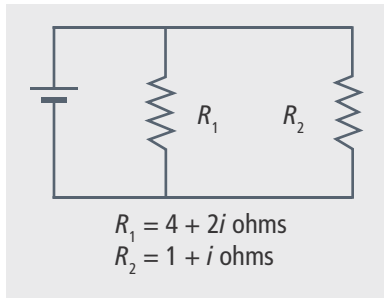
a. Find the total resistance. Write your answer in the form $a + bi$.

b. Show that the total resistance is equivalent to the expression $\frac{R_1 + R_2}{R_1 R_2}$.

c. Change the value of R_2 so that the total resistance is a real number. Explain how you chose the value.

47. Use Structure The complex number $a + bi$ can be represented on a coordinate plane as the point (a, b). You can use multiplication by i to rotate a point about the origin in the coordinate plane.
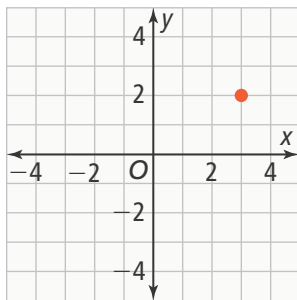
a. Write the point (x, y) on the graph as the complex number $x + yi$.

b. Multiply the complex number by i. Interpret the new value as a new point in the plane.

c. Repeat the steps above for two other points. How does multiplication by i rotate a point?

☑ **ASSESSMENT PRACTICE**

48. Complete the table by classifying each number as real, imaginary, or complex. Use the most specific classification. For example, all real numbers are also complex numbers, so it is more specific to classify a number as real.

Number	R, I, C
$2 + i$	C
$5 - 0i$	R
$2i$	I
$(3 - i)^2$	
$i^2 + 1$	
$3i$	
$(3 - i)(3 + i)$	
$(3 + i) - (2 + i)$	
$\sqrt{-14}$	
$i(4 + i) - 3i$	

49. SAT/ACT Which of the following is a solution to the equation $3x^2 = -12$?

Ⓐ $-4i$ Ⓑ $-2i$ Ⓒ -2 Ⓓ 2 Ⓔ $4i$

50. Performance Task Abby wants to write the square root of i in the form $a + bi$. She begins by writing the equation $\sqrt{i} = a + bi$.

Part A Square both sides of the equation. Then use the fact that the real part and imaginary part on each side of the equation are equal to write a system of equations involving the variables a and b.

Part B Solve the system to find b. Then find a.

Part C List the possible solutions for a and b.

Part D Square each of the possible solutions. What are the two square roots of i?