## UNDERSTAND

13. Make Sense and Persevere The table shows some values of a polynomial function. Deshawn says there are turning points between the $x$-values -3 and -2 and between 0 and 1 . He also says there is a relative minimum between the $x$-values -3 and -2 , and a relative maximum between 0 and 1 . Sketch a graph that shows how Deshawn could be correct and another graph that shows how Deshawn could be incorrect.

| $x$ | -5 | -4 | -3 | -2 | -1 | 0 | 1 | 2 |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\boldsymbol{f}(x)$ | -1004 | 129 | 220 | 85 | 12 | 1 | 4 | 165 |

14. Higher Order Thinking Use the information below to write a possible polynomial function. Explain how you determined your function and graph it to verify that it satisfies the criteria.

- 6 terms
- $y$-intercept at 1
- end behavior: As $x \rightarrow-\infty, y \rightarrow+\infty$.

$$
\text { As } x \rightarrow+\infty, y \rightarrow-\infty
$$

15. Reason An analyst for a new company used the first three years of revenue data to project future revenue for the company. The analyst predicts the function $f(x)=-2 x^{5}+6 x^{4}-x^{3}+5 x^{2}+6 x+50$ will give the revenue after $x$ years. Should the CEO expect the company to be successful? Explain.
16. Look for Relationships Sketch a graph of each of the functions described below.

- a cubic function with one $x$-intercept
- a cubic function with $2 x$-intercepts
- a cubic function with $3 x$-intercepts

17. Make Sense and Persevere Compare the rate of change for the function $f(x)=x^{3}-2 x^{2}+x+1$ over the intervals $[0,2]$ and $[2,4]$.

## PRACTICE

Write each polynomial function in standard form. For each function, find the degree, number of terms, and leading coefficient.
SEE EXAMPLE 1
18. $f(x)=-3 x^{3}+2 x^{5}+x+8 x^{3}-6+x^{4}-3 x^{2}$
19. $f(x)=8 x^{2}+10 x^{7}-7 x^{3}-x^{4}$
20. $f(x)=-x^{3}+9 x+12-x^{4}+5 x^{2}$

Use the leading coefficient and degree of the polynomial function to determine the end behavior of the graph. SEE EXAMPLE 2
21. $f(x)=-x^{5}+2 x^{4}+3 x^{3}+2 x^{2}-8 x+9$
22. $f(x)=7 x^{4}-4 x^{3}+7 x^{2}+10 x-15$
23. $f(x)=-x^{6}+7 x^{5}-x^{4}+2 x^{3}+9 x^{2}-8 x-2$

Use a table of values to estimate the intercepts and turning points of the function. Then graph the function. SEE EXAMPLE 3
24. $f(x)=x^{3}+2 x^{2}-5 x-6$
25. $f(x)=x^{4}-x^{3}-21 x^{2}+x+20$
26. Use the information below to sketch a graph of the polynomial function $y=f(x)$. See EXAMPLE 4

- $f(x)$ is positive on the intervals $(-\infty,-3),(-2,0)$, and $(2,3)$.
- $f(x)$ is negative on the intervals $(-3,-2),(0,2)$, and $(3, \infty)$.
- $f(x)$ is increasing on the interval $(-2.67,-1)$ and (1, 2.5).
- $f(x)$ is decreasing on the intervals $(-\infty,-2.67)$, $(-1,1)$, and $(2.5, \infty)$.

27. The equation shown models the average depth $y$, in feet, of a lake, $x$ years after 2016, where $0<x<6$. Use technology to graph the function. In what year does this model predict a relative minimum value for the depth? SEE EXAMPLE 5


## APPLY

28. Use Structure Allie has a piece of construction paper that she wants to use to make an open rectangular prism. She will cut a square with side length $x$ from each corner of the paper, so the length and width is decreased by $2 x$ as shown in the diagram.

a. Write a function that models the volume of the rectangular prism.
b. Graph the function and identify a reasonable domain.
c. What do the $x$-intercepts of the graph mean in this context?
d. If Allie wants to maximize the volume of the box, what is the side length of the squares that should be cut from each corner of the piece of construction paper? Explain.
29. Make Sense and Persevere Alberto is designing a container in the shape of a rectangular prism to ship electronic devices. The length of the container is 10 inches longer than the height. The sum of the length, width, and height is 25 inches. The volume of the container, in terms of height $x$, is shown. Use a graphing calculator to graph the function. What do the $x$-intercepts of the graph mean in this context? What dimensions of the container will maximize the volume?


## ASSESSMENT PRACTICE

30. Copy and complete the table to give the leading coefficient of each polynomial function.

| Polynomial Function | Leading <br> coefficient |
| :--- | :--- |
| $f(x)=3 x^{-3}+2 x^{2}+9 x-6$ |  |
| $f(x)=-3 x^{4}+8 x^{2}-2 x+7 x^{5}$ |  |
| $f(x)=-3 x^{5}+7 x^{3}+6 x^{2}-2$ |  |
| $f(x)=3 x^{2}-12 x^{4}-3 x^{6}-3 x^{3}$ |  |
| $f(x)=6 x^{3}+9 x^{2}-5 x-3$ |  |

31. SAT/ACT What is the maximum number of terms a fourth-degree polynomial function in standard form can have?
(A) 1
(B) 2
(C) 3
(D) 4
(E) 5
32. Performance Task In the year 2000, a demographer predicted the estimated population of a city, which can be modeled by the function $f(x)=5 x^{4}-4 x^{3}+25 x+8,000$. Several years later, a statistician, using data from the U.S. Census Bureau, modeled the actual population with the function $P(x)=7 x^{4}-6 x^{3}+5 x+8,000$. The graphs of the functions are shown.


Part A What is the $y$-intercept of each function, and what does it represent?

Part B Identify the end behaviors of $f$ and $P$.
Part C Compare the average rates of change of $f$ and $P$ from 2003 to 2005.

