



UNDERSTAND

- Generalize** Explain two methods by which $(2m^3 + 4n^2)^2$ can be simplified. Which method do you prefer and why?
- Use Structure** Polynomial function P is the sum of two polynomial functions, one with degree 2 and a positive leading coefficient and one with degree 3 and a negative leading coefficient. Describe the end behavior of P . Write an example of two polynomial functions and their sum, P , to justify your description.
- Generalize** Multiply the polynomials $(a + b)$ $(a + b)(a + b)$ to develop a general formula for cubing a binomial, $(a + b)^3$.
- Reason** Polynomial function R is the difference of two degree-two polynomial functions. What are the possible degrees for R ? Explain.
- Error Analysis** Describe and correct the error a student made in multiplying the polynomials.

$$\begin{aligned}
 &(y - 2)(3y^2 - y - 7) \\
 &= y(3y^2 - y - 7) - 2(3y^2 - y - 7) \\
 &= 3y^3 - y^2 - 7y + (-6y^2) + (-2y) - 14 \\
 &= 3y^3 - 7y^2 - 9y - 14
 \end{aligned}$$



- Higher Order Thinking** Do you think polynomials are closed under division? Explain why you think so, or provide a counterexample.
- Construct Arguments** Explain why the expression $9x^3 + \frac{1}{2}x^2 + 3x^{-1}$ is not a polynomial.
- Communicate Precisely** Explain the difference between the graphs of polynomial functions with a degree of 3 that have a positive leading coefficient and the graphs of those with a negative leading coefficient.

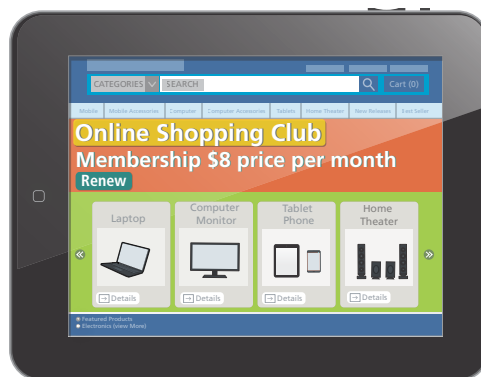
PRACTICE

Add or subtract the polynomials. SEE EXAMPLE 1

- $(2x^3 + 3x^2 + 4) + (6x^3 - x^2 - 5x)$
- $(5y^4 + 3y^3 - 6y^2 + 14) - (-y^4 + y^2 - 7y - 1)$
- $(4p^2q^2 + 2p^2q - 7pq) - (9p^2q^2 + 5pq^2 - 11pq)$

Multiply the polynomials. SEE EXAMPLE 2

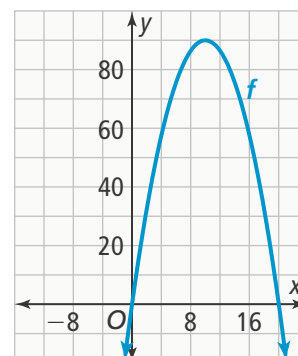
- $-4xy(5x^2 - 9xy - y^2)$
- $(3c - 4)(2c^2 - 5c + 7)$
- $(z + 5)(z - 9)(1 - z)$
- Is the set of monomials closed under addition? Explain why you think so, or provide a counterexample. SEE EXAMPLE 3
- An online shopping club has 13,500 members when it charges \$8 per month for membership. For each \$1 monthly increase in membership fee, the club loses approximately 500 of its existing members.



Write and simplify a function R to represent the monthly revenue received by the club when x represents the price increase.

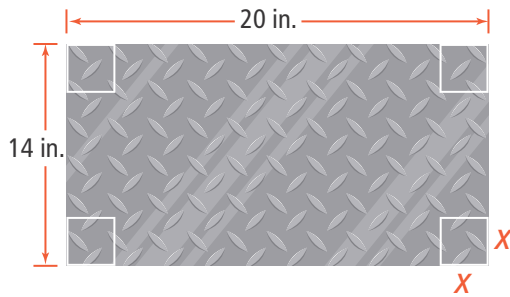
Hint Monthly revenue = # members • monthly fee SEE EXAMPLE 4

- The graph shows a polynomial function f . Polynomial function $g = x^2(6 - x)$. Compare the maximum values and the end behavior of the functions f and g when $x > 0$. SEE EXAMPLE 5

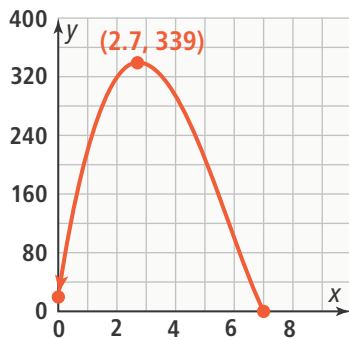


APPLY

Use this information for 27 and 28. A foundry manufactures aluminum trays from pieces of sheet metal as shown.



27. **Model With Mathematics** Let x represent the side length of each square.
- Write expressions for the length, width, and height of the metal tray.
 - Write and simplify a polynomial function V to represent the volume of the tray.
 - Using the graph of the function V , explain what the marked vertex represents.



28. **Reason** Suppose the foundry manufacturer has a new design where the squares cut from the corners have sides that are half the length of the squares in the previous design.
- Write expressions for the length, width, and height of this tray.
 - Write and simplify the polynomial function $v(x)$, to represent the volume of the new tray.
 - Write the function $D(x)$ that represents the difference, $V(x) - v(x)$.
29. **Make Sense and Persevere** Jacy has \$1,000 to invest in a fund that pays approximately 4.6% per year or in a savings account with an annual interest rate of 1.8%. Write a polynomial function $S(x)$ to represent the interest Jacy will earn in 1 year by investing x dollars in the fund and the remainder in the savings account.

ASSESSMENT PRACTICE

30. Are polynomials open or closed under each operation? Classify each operation as *open* or *closed*.
- addition
 - subtraction
 - multiplication
 - division
31. **SAT/ACT** Which of the following functions is NOT a polynomial function?
- $2y^2 + 9y - 8$
 - $-\frac{1}{2}x^3 + 8$
 - $(x - 1)(5 - x)(x + 4)$
 - $9z^4 + 2z + \frac{1}{2}$
32. **Performance Task** Consider the polynomial functions $P(x) = x^2 - 4$ and $R(x) = -x^2 - 2x$.

Part A Write and simplify a polynomial function $T(x)$ that is the product of P and R .

Part B Copy and complete the table of values for all three functions.

| x | $P(x)$ | $R(x)$ | $T(x)$ |
|-----|--------|--------|--------|
| -3 | | | |
| -2 | | | |
| -1 | | | |
| 0 | | | |
| 1 | | | |
| 2 | | | |
| 3 | | | |

Part C Graph the functions on the same coordinate grid.

Part D How do the zeros of T relate to the zeros of P and R ?

Part E Explain how you can identify the intervals in which T is positive by analyzing the R and P .