



**UNDERSTAND**

- Reason** Write a polynomial division problem with a quotient of  $x^2 - 5x + 7$  and a remainder of 2. Explain your reasoning. How can you verify your answer?
- Communicate Precisely** Show that  $x - 3$  and  $x + 5$  are factors of  $x^4 + 2x^3 - 16x^2 - 2x + 15$ . Explain your reasoning.
- Error Analysis** Alicia divided the polynomial  $2x^3 - 4x^2 + 6x + 10$  by  $x^2 + x$ . Describe and correct the error Alicia made in dividing the polynomials.

$$2x - 6 + \frac{10}{x^2 + x}$$

$$\begin{array}{r} x^2 + x \overline{) 2x^3 - 4x^2 + 6x + 10} \\ \underline{-(2x^3 + 2x^2)} \phantom{+ 10} \\ -6x^2 + 6x \phantom{+ 10} \\ \underline{-(-6x^2 - 6x)} \phantom{+ 10} \\ 10 \end{array}$$

- Higher Order Thinking** When dividing polynomial  $P(x)$  by polynomial  $d(x)$ , the remainder is  $R(x)$ . The remainder can also be written as  $\frac{R(x)}{d(x)}$ . How can you use the degrees of  $R(x)$  and  $d(x)$  to determine you are finished dividing?
- Look for Relationships** When dividing polynomial  $P(x)$  by polynomial  $x - n$ , the remainder is 0. When graphing  $P(x)$ , what is an  $x$ -intercept of the graph?
- Reason** When dividing  $x^3 + nx^2 + 4nx - 6$  by  $x + 3$ , the remainder is  $-48$ . What is the value of  $n$ ?
- Mathematical Connections** Use polynomial long division to divide  $8x^3 + 27$  by  $2x + 3$ . How can you use multiplication to check your answer? Show your work.

**PRACTICE**

**Use long division to divide.** SEE EXAMPLE 1

- $x^3 + 5x^2 - x - 5$  divided by  $x - 1$
- $2x^3 + 9x^2 + 10x + 3$  divided by  $2x + 1$
- $3x^3 - 2x^2 + 7x + 9$  divided by  $x^2 - 3x$
- $2x^4 - 6x^2 + 3$  divided by  $2x - 6$

**Use synthetic division to divide.** SEE EXAMPLE 2

- $x^4 - 25x^2 + 144$  divided by  $x - 4$
- $x^3 + 6x^2 + 3x - 10$  divided by  $x + 5$
- $x^5 + 2x^4 - 3x^3 + x - 1$  divided by  $x + 2$
- $-x^4 + 7x^3 + x^2 - 2x - 12$  divided by  $x - 3$
- Use synthetic division to show that the remainder of  $f(x) = x^4 - 6x^3 - 33x^2 + 46x + 75$  divided by  $x - 9$  is  $P(9)$ . SEE EXAMPLE 3

**Use the Remainder Theorem to evaluate each polynomial for the given value of  $x$ .** SEE EXAMPLE 4

- $f(x) = x^3 + 9x^2 + 3x - 7$ ;  $x = -5$
- $f(x) = 2x^3 - 3x^2 + 4x + 13$ ;  $x = 3$
- $f(x) = -x^4 + 2x^3 - x^2 + 4x + 8$ ;  $x = -2$
- $f(x) = x^5 - 3x^4 - 2x^3 + x^2 - 2x - 1$ ;  $x = 4$

**Is each given binomial a factor of the given polynomial? If so, write the polynomial as a product of two factors.** SEE EXAMPLE 5

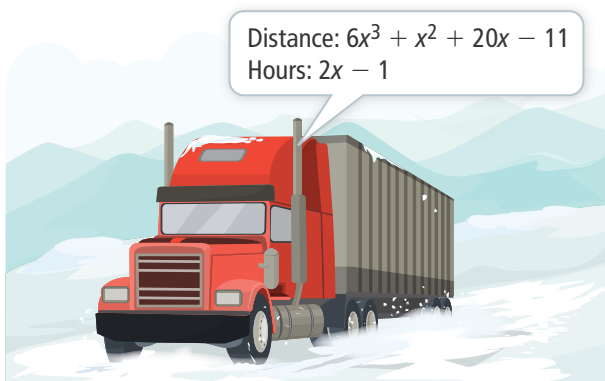
- polynomial:  $P(x) = 8x^3 - 10x^2 + 28x - 16$ ; binomial:  $x - 3$
- polynomial:  $P(x) = 4x^4 - 9x^3 - 7x^2 - 2x + 25$ ; binomial:  $x + 4$
- polynomial:  $P(x) = -x^5 + 12x^3 + 6x^2 - 23x + 1$ ; binomial:  $x - 2$
- polynomial:  $P(x) = 2x^3 + 3x^2 - 8x - 12$ ; binomial:  $2x + 3$

**APPLY**

32. **Model With Mathematics** Darren is placing shipping boxes in a storage unit with a floor area of  $x^4 + 5x^3 + x^2 - 20x - 14$  square units. Each box has a volume of  $x^3 + 10x^2 + 29x + 20$  cubic units and can hold a stack of items with a height of  $x + 5$  units.



- How much floor space will each box cover?
  - What is the maximum number of boxes Darren can place on the floor of the storage unit?
  - Assume Darren places the maximum number of boxes on the floor of the storage unit, with no overlap. How much of the floor space is not covered by a box?
33. **Reason** Lauren wants to determine the length and height of her DVD stand. The function  $f(x) = x^3 + 14x^2 + 57x + 72$  represents the volume of the DVD stand, where the width is  $x + 3$  units. What are possible dimensions for the length and height of the DVD stand? Explain.
34. **Make Sense and Persevere** A truck traveled  $6x^3 + x^2 + 20x - 11$  miles in  $2x - 1$  hours. At what rate did the semi-truck travel? (*Hint:* Use the formula  $d = rt$ , where  $d$  is the distance,  $r$  is the rate, and  $t$  is the time.)



**ASSESSMENT PRACTICE**

35. When polynomial  $P(x)$  is divided by the linear factor  $x - n$ , the remainder is 0. What can you conclude? Select all that apply.
- $P(x) = 0$
  - $P(n) = 0$
  - $P(-n) = 0$
  - $x - n$  is a factor of  $P(x)$ .
  - $x + n$  is a factor of  $P(x)$ .
36. **SAT/ACT**  $x + 3$  is a factor of the polynomial  $x^3 + 2x^2 - 5x + n$ . What is the value of  $n$ ?
- 6
  - 3
  - 2
  - 3
  - 6
37. **Performance Task** The table shows some quotients of the polynomial  $x^n - 1$  divided by the linear factor  $x - 1$ .

Dividend	Divisor	Quotient
$x^2 - 1$	$x - 1$	$x + 1$
$x^3 - 1$	$x - 1$	$x^2 + x + 1$
$x^4 - 1$	$x - 1$	
$x^5 - 1$	$x - 1$	
$x^6 - 1$	$x - 1$	

**Part A** Use long division or synthetic division to find the missing quotients to complete the table.

**Part B** Look for a pattern. Then describe the pattern when  $x^n - 1$  is divided by  $x - 1$ .

**Part C** Use the pattern to find the quotient when  $x^{10} - 1$  is divided by  $x - 1$ .