PRACTICE & PROBLEM SOLVING

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Practice Tutorial

Additional Exercises Available Online

UNDERSTAND

13. Construct Arguments Consider the polynomial $P(x) = 5x^3 + mx^2 + nx + 6$, where m and n are rational coefficients. Is 3 *sometimes, always,* or *never* a root? Explain.

14. Use Structure Write a fourth-degree polynomial function Q with roots -1, 0, and $2i$.

15. Error Analysis A student says that a fifth-degree polynomial equation with rational coefficients has roots -5, -3, 1, 2, and $\sqrt{3}$. Describe possible errors the student may have made.

16. Reason Write a third-degree polynomial with rational coefficients that has the following possible roots. Explain your reasoning.

$$\pm\frac{1}{1}, \pm\frac{1}{2}, \pm\frac{2}{1}, \pm\frac{2}{2}, \pm\frac{5}{1}, \pm\frac{5}{2}, \pm\frac{10}{1}, \pm\frac{10}{2}$$

17. Error Analysis Describe and correct the error a student made in finding the roots of the polynomial equation $2x^3 - x^2 - 10x + 5 = 0$.

List all possible rational roots.

$\pm 1, \pm\frac{1}{2}, \pm 5, \pm\frac{5}{2}$

Testing reveals that $\frac{1}{2}$ is a root.

Dividing the polynomial by the binomial $x - \frac{1}{2}$ results in the factored form

$f(x) = (x - \frac{1}{2})(2x^2 - 10)$

The equation $2x^2 - 10 = 0$ has two irrational roots, $\sqrt{10}$ and $-\sqrt{10}$.

The complete set of roots is $\{\frac{1}{2}, \sqrt{10}, -\sqrt{10}\}$.

✗

18. Higher Order Thinking What is the least number of terms a fifth-degree polynomial with root $3i$ can have? Give an example of such a polynomial equation. Explain.

19. Use Structure Show that the Fundamental Theorem of Algebra is true for all quadratic equations with real coefficients. (*Hint*: Use the Quadratic Formula and examine the possibilities for the value of the discriminant.)

PRACTICE

List all the possible rational solutions for each equation. SEE EXAMPLE 1

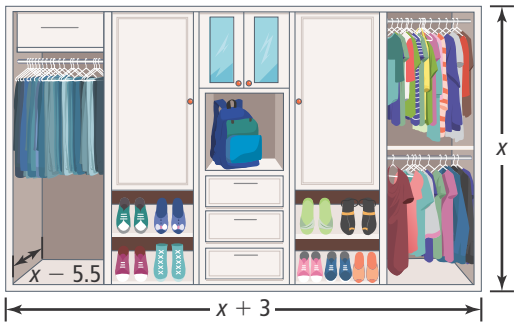
20. $0 = x^3 - 3x^2 + 4x - 12$

21. $0 = 2x^4 + 13x^3 - 47x^2 - 13x + 45$

22. $0 = 4x^3 + 64x^2 - x - 16$

23. $0 = 8x^3 + 11x^2 - 13x - 6$

24. A closet in the shape of a rectangular prism has the measurements shown. What is the height of the closet, in feet, if its volume is 220 ft^3? SEE EXAMPLE 2



What are all real and complex roots of the following functions? SEE EXAMPLE 3

25. $0 = x^3 - 3x - 52$

26. $0 = x^3 + 9x^2 - 7x - 63$

27. $0 = x^4 + 34x^2 - 72$

28. $0 = x^6 + 4x^4 - 41x^2 + 36$

29. Suppose a cubic polynomial f has one rational zero c and two irrational zeros which are a conjugate pair $a + \sqrt{b}$ and $a - \sqrt{b}$, where a and b are rational numbers. Does f have rational coefficients? SEE EXAMPLE 4

Find a polynomial function $P(x)$ such that P has the degree and $P(x) = 0$ has the root(s) listed. SEE EXAMPLE 5

30. degree of $P = 2$;
zero: $1 + 6i$

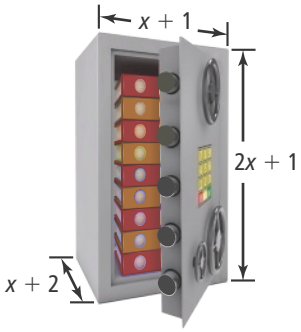
31. degree of $P = 4$;
zeros: $3 - \sqrt{11}$ and $-9i$

32. degree of $P = 3$;
zeros: -5 and $4 - 8i$

PRACTICE & PROBLEM SOLVING

APPLY

33. Make Sense and Persevere A fireproof safe has the measurements shown.



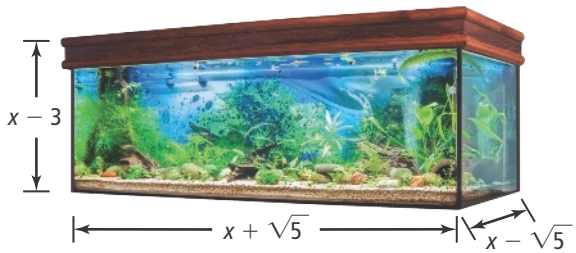
a. Write an equation to represent the situation when the volume of the fireproof safe is 270 in.^{3}. Rewrite the equation in the form $P(x) = 0$.

b. List all of the possible factors of the polynomial expression.

c. What are the real roots of the equation? Explain how you know these are the only real roots.

d. What are the length, width, and height of the fireproof safe?

34. Make Sense and Persevere What are the dimensions of the fish tank, in feet, if its volume is 176 ft^3?



35. Reason The cost of producing x video game consoles is modeled by the function $C(x) = x^4 - 5x^3 - 12x^2 - 22x - 40$. If a company spent $1,706 to produce video game consoles, how many consoles were made?

ASSESSMENT PRACTICE

36. A fifth-degree polynomial equation with rational coefficients has the roots 3, $8i$, and $7 - \sqrt{5}$. Which are also roots of the polynomial equation? Select all that apply.

Ⓐ -3
Ⓑ $-8i$
Ⓒ $1 - 8i$
Ⓓ $-7 - \sqrt{5}$
Ⓔ $7 + \sqrt{5}$

37. SAT/ACT Which is a third-degree polynomial equation with rational coefficients that has roots -2 and $6i$?

Ⓐ $x^3 + 2x^2 + 36x + 72$
Ⓑ $x^3 - 2x^2 + 36x - 72$
Ⓒ $x^3 + 2x^2 - 36x - 72$
Ⓓ $x^2 + (6i - 2)x - 12$
Ⓔ $x^2 - (6i - 2)x - 12$

38. Performance Task The table shows the number of possible real and imaginary roots for an nth degree polynomial equation with rational coefficients.

Degree	Real Roots	Imaginary Roots
3	3	0
3	1	2
5	5	0
5	3	2
5	1	4

Part A List all of the possible combinations of real and imaginary roots for a seventh-degree polynomial equation.

Part B What do you notice about the number of real roots of a polynomial equation with an odd degree?