TOPIC 3

Topic Review

TOPIC ESSENTIAL QUESTION

1. What can the rule for a polynomial function reveal about its graph, and what can the graphs of polynomial functions reveal about the solutions of polynomial equations?

Vocabulary Review

Choose the correct term to complete each sentence.

2. The __________ is the greatest power of the variable in a polynomial expression.
3. The __________ is the non-zero constant multiplied by the greatest power of the variable in a polynomial expression.
4. The __________ of a function describes what happens to its graph as x approaches positive and negative infinity.
5. __________ is the triangular pattern of numbers where each number is the sum of two numbers above it.
6. The __________ determines whether the graph of the function will cross the x-axis at the point or merely touch it.
7. The __________ is a formula that can be used to expand powers of binomial expressions.
8. __________ is a method to divide a polynomial by a linear factor whose leading coefficient is 1.

- Binomial Theorem
- degree of a polynomial
- end behavior
- even function
- Factor Theorem
- identity
- leading coefficient
- multiplicity of a zero
- Pascal's Triangle
- synthetic division

Concepts & Skills Review

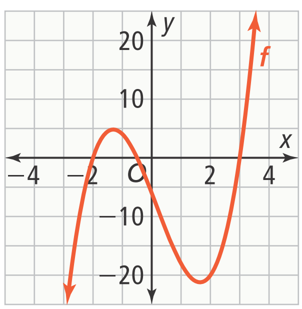
LESSON 3-1 Graphing Polynomial Functions

Quick Review

A **polynomial** can be either a monomial or a sum of monomials. When a polynomial has more than one monomial, the monomials are also referred to as **terms.**

Example

Graph the function $f(x) = 2x^3 - x^2 - 13x - 6$.



There are zeros at $x = -2$, $x = -0.5$, and $x = 3$.

There are turning points between –2 and –0.5 and between –0.5 and 3.

As $x \to -\infty$, $y \to -\infty$.

As $x \to +\infty$, $y \to +\infty$.

Practice & Problem Solving

Graph the polynomial function. Estimate the zeros and the turning points of the graph.

9. $f(x) = x^5 + 2x^4 - 10x^3 - 20x^2 + 9x + 18$

10. $f(x) = x^4 + x^3 - 16x^2 - 4x + 48$

11. **Reason** A polynomial function has the following end behavior: As $x \to -\infty$, $y \to +\infty$. As $x \to +\infty$, $y \to -\infty$. Describe the degree and leading coefficient of the polynomial function.

12. **Make Sense and Persevere** After x hours of hiking, Sadie's elevation is $p(x) = -x^3 + 11x^2 - 34x + 24$, in meters. After how many hours will Sadie's elevation be 18 m below sea level? What do the x- and y-intercepts of the graph mean in this context?

TOPIC 3 REVIEW

LESSONS 3-2 & 3-3 Adding, Subtracting, and Multiplying Polynomials and Polynomial Identities

Quick Review

To add or subtract polynomials, add or subtract like terms. To multiply polynomials, use the Distributive Property.

Polynomial identities can be used to factor or multiply polynomials.

Example

Add $(-2x^3 + 5x^2 + 2x - 3) + (x^3 - 6x^2 + x + 12)$.

Use the Commutative and Associative Properties. Then combine like terms.

$(-2x^3 + 5x^2 + 2x - 3) + (x^3 - 6x^2 + x - 12)$

$= (-2x^3 + x^3) + (5x^2 - 6x^2) + (2x + x) + (-3 + 12)$

$= -x^3 - x^2 + 3x + 9$

Example

Use polynomial identities to factor $8x^3 + 27y^3$.

Use the Sum of Cubes Identity. Express each term as a square. Then write the factors.

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$8x^3 + 27y^3 = (2x)^3 + (3y)^3$

$= (2x + 3y)(4x^2 - 6xy + 9y^2)$

Practice & Problem Solving

Add or subtract the polynomials.

13. $(-8x^3 + 7x^2 + x - 9) + (5x^3 + 3x^2 - 2x - 1)$

14. $(9y^4 - y^3 + 4y^2 + y - 2) - (2y^4 - 3y^3 + 6y - 7)$

Multiply the polynomials.

15. $(9x - 1)(x + 5)(7x + 2)$

Use polynomial identities to multiply each polynomial.

16. $(5x + 8)^2$

17. $(7x - 4)(7x + 4)$

Factor the polynomial.

18. $x^6 - 64$

19. $27x^3 + y^6$

Use Pascal's Triangle or the Binomial Theorem to expand the expressions.

20. $(x - 2)^4$

21. $(x + 5y)^5$

22. **Communicate Precisely** Explain why the set of polynomials is closed under subtraction.

23. **Reason** The length of a rectangle is represented by $3x^3 - 2x^2 + 10x - 4$, and the width is represented by $-x^3 + 6x^2 - x + 8$. What is the perimeter of the rectangle?

LESSON 3-4 Dividing Polynomials

Quick Review

Polynomials can be divided using long division or synthetic division. **Synthetic division** is a method to divide a polynomial by a linear factor whose leading coefficient is 1.

Example

Use synthetic division to divide $x^4 - 5x^3 - 6x^2 + 2x - 8$ by $x + 3$.

$$\begin{array}{r|rrrrr} -3 & 1 & -5 & -6 & 2 & -8 \\ & & -3 & 24 & -54 & 156 \\ \hline & 1 & -8 & 18 & -52 & 148 \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ & x^3 & -8x^2 & +18x & -52 & +\frac{148}{x+3} \end{array}$$

The quotient is $x^3 - 8x^2 + 18x - 52$, and the remainder is 148.

Practice & Problem Solving

Use long division to divide.

24. $x^4 + 2x^3 - 8x^2 - 3x + 1$ divided by $x + 2$

Use synthetic division to divide.

25. $x^4 + 5x^3 + 7x^2 - 2x + 17$ divided by $x - 3$

26. **Make Sense and Persevere** A student divided $f(x) = x^3 + 8x^2 - 9x - 3$ by $x - 2$ and got a remainder of 19. Explain how the student could verify the remainder is correct.

27. **Reason** The area of a rectangle is $4x^3 + 14x^2 - 18$ in.^{2}. The length of the rectangle is $x + 3$ in. What is the width of the rectangle?

LESSONS 3-5 & 3-6 Zeros of Polynomial Functions and Theorems about Roots of Polynomial Equations

Quick Review

You can factor and use synthetic division to find zeros of polynomial functions. Then you can use the zeros to sketch a graph of the function.

The **Rational Root Theorem** states that the possible rational roots, or zeros, of a polynomial equation with integer coefficients come from the list of numbers of the form: $\pm\frac{\text{factor of } a_0}{\text{factor of } a_n}$.

Example

List all the possible rational solutions for the equation $0 = 2x^3 + x^2 - 7x - 6$. Then find all of the rational roots.

$\pm1, \pm2, \pm3, \pm6$ Factors of the constant term

$\pm1, \pm2$ Factors of the leading coefficient

List the possible roots, eliminating duplicates.

$\pm\frac{1}{1}, \pm\frac{1}{2}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{3}{2}, \pm\frac{6}{1}$

Use synthetic division to find that the roots are $-\frac{3}{2}$, -1, and 2.

Practice & Problem Solving

Sketch the graph of the function.

28. $f(x) = 2x^4 - x^3 - 32x^2 + 31x + 60$

29. $g(x) = x^3 - x^2 - 20x$

30. What x-values are solutions to the equation $x^3 + 2x^2 - 4x + 8 = x^2 - x + 4$?

31. What values of x are solutions to the inequality $x^3 + 3x^2 - 4x - 12 > 0$?

32. What are all of the real and complex roots of the function $f(x) = x^4 - 4x^3 + 4x^2 - 36x - 45$?

33. A polynomial function Q of degree 4 with rational coefficients has zeros $1 + \sqrt{5}$ and $-7i$. What is an equation for Q?

34. **Reason** What does the graph of a function tell you about the multiplicity of a zero?

35. **Make Sense and Persevere** A storage unit in the shape of a rectangular prism measures $2x$ ft long, $x + 8$ ft wide, and $x + 9$ ft tall. What are the dimensions of the storage unit, in feet, if its volume is 792 ft^3?

LESSON 3-7 Transformations of Polynomial Functions

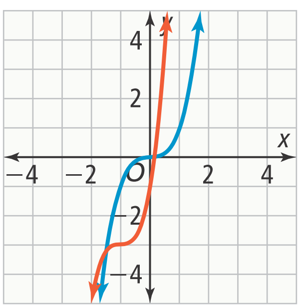
Quick Review

Polynomial functions can be translated, reflected, and stretched in similar ways to other functions you have studied.

Example

How does the graph of $f(x) = 2(x + 1)^3 - 3$ compare to the graph of the parent function?

Parent function: $y = x^3$



Adding 1 shifts the graph to the left 1 unit.

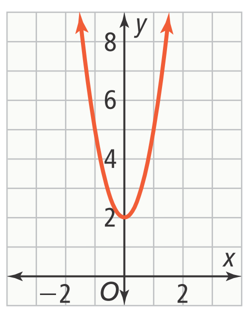
Multiplying by 2 stretches the graph vertically.

Subtracting 3 shifts the graph down 3 units.

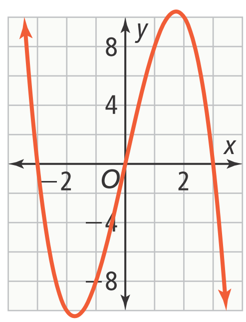
Practice & Problem Solving

Classify each function as even, odd, or neither.

36.



37.



38. **Error Analysis** A student says the graph of $f(x) = 0.5x^4 + 1$ is a vertical stretch and a translation up 1 unit of the parent function. Explain the student's error.

39. **Make Sense and Persevere** The volume of a refrigerator, in cubic centimeters, is given by the function $V(x) = (x)(x + 1)(x - 2)$. Write a new function for the volume of the refrigerator in cubic millimeters if x is in centimeters.