TOPIC 4

Topic Review

TOPIC ESSENTIAL QUESTION

1. How do you calculate with functions defined as quotients of polynomials, and what are the key features of their graphs?

Vocabulary Review

Choose the correct term to complete each sentence.

2. The ___________ can be represented by the equation $y = \frac{1}{x}$.
3. A(n) ___________ is any function $R(x) = \frac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomials and $Q(x) \neq 0$.
4. ___________ can be modeled by the equation $y = \frac{k}{x}$.
5. A(n) ___________ is the quotient of two polynomials.
6. A(n) ___________ is a line that a graph approaches but may not touch.
7. A(n) ___________ is a fraction that has one or more fractions in the numerator and/or the denominator.
8. A(n) ___________ is a value that is a solution to an equation that is derived from an original equation but does not satisfy the original equation.

- inverse variation
- constant of variation
- reciprocal function
- asymptote
- rational function
- extraneous solution
- rational expression
- compound fraction

Concepts & Skills Review

LESSON 4-1 Inverse Variation and the Reciprocal Function

Quick Review

The equation $y = \frac{k}{x}$, or $xy = k$, $k \neq 0$, represents an **inverse variation**, where k is the **constant of variation**. The parent **reciprocal function** is $y = \frac{1}{x}$.

Example

In an inverse variation, $x = 9$ when $y = 2$. What is the value of y when $x = 3$?

$2 = \frac{k}{9}$ ········ Substitute 9 and 2 for x and y.

$18 = k$ ········ Solve for k.

$y = \frac{18}{3}$ ········ Substitute 18 and 3 for k and x, respectively.

$y = 6$ ········ Divide.

Practice & Problem Solving

9. In an inverse variation, $x = 2$ when $y = -4$. What is the value of y when $x = 16$?
10. In an inverse variation, $x = 6$ when $y = \frac{1}{12}$. What is the value of x when $y = 2$?
11. Graph the function $y = \frac{5}{x}$. What are the domain, range, and asymptotes of the function?
12. **Look for Relationships** How is the parent reciprocal function related to an inverse variation?
13. **Make Sense and Persevere** The volume, V, of a gas varies inversely with pressure, P. If the volume of a gas is 6 cm^3 with pressure 25 kg/cm^2, what is the volume of a gas with pressure 15 kg/cm^2?

TOPIC 4 REVIEW

LESSON 4-2 Graphing Rational Functions

Quick Review

Vertical asymptotes may occur when the denominator of a rational function is equal to 0.

Horizontal asymptotes guide the end behavior of a graph and depend on the degrees of the numerator and denominator.

Example

What is the graph of $f(x) = \frac{9x^2 - 25}{x^2 - 5x - 6}$?

Find vertical asymptotes.

$x^2 - 5x - 6 = 0$ ········· Set denominator equal to 0.

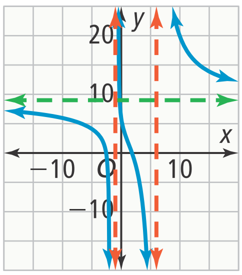
$(x + 1)(x - 6) = 0$ ········· Factor.

$x = -1 \quad x = 6$ ········· Solve.

Find horizontal asymptotes.

Find the ratio of leading terms.

$f(x) = \frac{9x^2}{x^2} = 9$



Practice & Problem Solving

Identify the vertical and horizontal asymptotes of each rational function.

14. $f(x) = \frac{x - 8}{x^2 + 9x + 14}$

15. $f(x) = \frac{2x + 1}{x^2 + 5x - 6}$

16. $f(x) = \frac{x^2 - 9}{2x^2 + 25}$

17. $f(x) = \frac{16x^2 - 1}{x^2 - 6x - 16}$

Graph each function and identify the horizontal and vertical asymptotes.

18. $f(x) = \frac{x}{x^2 - 1}$

19. $f(x) = \frac{3}{x - 2}$

20. $f(x) = \frac{2x^2 + 7}{x^2 + 2x + 1}$

21. $f(x) = \frac{3x^2 - 11x - 4}{4x^2 - 25}$

22. Reason The daily attendance at an amusement park after day x is given by the function $f(x) = \frac{3{,}000x}{x^2 - 1}$. On approximately which day will the attendance be 1,125 people?

LESSON 4-3 Multiplying and Dividing Rational Expressions

Quick Review

To multiply **rational expressions**, divide out common factors and simplify. To divide rational expressions, multiply by the reciprocal of the divisor.

Example

What is the quotient of $\frac{x^2 + x - 2}{x + 3}$ and $\frac{x^2 + 3x - 4}{2x + 6}$?

$= \frac{x^2 + x - 2}{x + 3} \cdot \frac{2x + 6}{x^2 + 3x - 4}$ ········· Multiply by reciprocal.

$= \frac{(x + 2)\cancel{(x - 1)}}{\cancel{x + 3}} \cdot \frac{2\cancel{(x + 3)}}{(x + 4)\cancel{(x - 1)}}$ ········· Divide out common factors.

$= \frac{2(x + 2)}{x + 4}$ ········· Simplify.

Practice & Problem Solving

Find the simplified product, and state the domain.

23. $\frac{x^2 + x - 12}{x^2 - x - 6} \cdot \frac{x + 2}{x + 4}$

24. $\frac{x^2 + 8x}{x^3 + 5x^2 - 24x} \cdot (x^3 + 2x^2 - 15x)$

Find the simplified quotient, and state the domain.

25. $\frac{x^2 - 36}{x^2 - 3x - 18} \div \frac{x^2 + 2x - 24}{x^2 + 7x + 12}$

26. $\frac{2x^2 + 5x - 3}{x^2 - 4x - 21} \div \frac{2x^2 + 5x - 3}{3x + 9}$

27. Reason The volume, in cubic units, of a rectangular prism with a square base can be represented by $25x^3 + 200x^2$. The height, in units, can be represented by $x + 8$. What is the side length of the base of the rectangular prism, in units?

LESSON 4-4 Adding and Subtracting Rational Expressions

Quick Review

To add or subtract rational expressions, multiply each expression in both the numerator and denominator by a common denominator. Add or subtract the numerators. Then simplify.

Example

What is the sum of $\frac{x-2}{x^2-25}$ and $\frac{3}{x+5}$?

$\frac{x-2}{(x+5)(x-5)} + \frac{3}{x+5}$ .... Factor denominators.

$= \frac{x-2}{(x+5)(x-5)} + \frac{3(x-5)}{(x+5)(x-5)}$ .... Find common denominator.

$= \frac{x-2+3(x-5)}{(x+5)(x-5)}$ .... Add numerators.

$= \frac{x-2+3x-15}{(x+5)(x-5)}$ .... Multiply.

$= \frac{4x-17}{(x+5)(x-5)}$ .... Simplify.

Practice & Problem Solving

Find the sum or difference.

28. $\frac{2x}{x+6} + \frac{3}{x-1}$

29. $\frac{x}{x^2-4} - \frac{5}{x-2}$

Simplify.

30. $\frac{2+\frac{2}{x}}{2-\frac{2}{x}}$

31. $\frac{\frac{-1}{x}+\frac{3}{y}}{\frac{4}{x}-\frac{5}{y}}$

32. Communicate Precisely Why is it necessary to consider the domain when adding and subtracting rational expressions?

33. Make Sense and Persevere Mia paddles a kayak 6 miles downstream at a rate 4 mph faster than the river's current. She then travels 6 miles back upstream at a rate 2 mph faster than the river's current. Write and simplify an expression for the time it takes her to make the round trip in terms of the river's current c.

LESSON 4-5 Solving Rational Equations

Quick Review

A **rational equation** is an equation relating rational expressions. An **extraneous solution** is a value that is a solution to an equation that is derived from an original equation but does not satisfy the original equation.

Example

What are the solutions to the equation $\frac{2}{x-2} = \frac{x}{x-2} - \frac{x}{4}$?

$(4)(x-2)\left(\frac{2}{x-2}\right)$

$= \left(\frac{x}{x-2} - \frac{x}{4}\right)(4)(x-2)$ .... Multiply by the LCD.

$8 = 4x - x^2 + 2x$ .... Multiply.

$x^2 - 6x + 8 = 0$ .... Write in standard form.

$(x-2)(x-4) = 0$ .... Factor.

$x-2=0$ or $x-4=0$ .... Zero Product Property

$x=2$ or $x=4$ .... Solve to identify possible solutions.

The solution $x = 2$ is extraneous. The only solution to the equation is $x = 4$.

Practice & Problem Solving

Solve the equation.

34. $\frac{18}{x+4} = 6$

35. $\frac{9}{x-1} = 3$

36. $-\frac{4}{3} + \frac{2}{x} = 8$

37. $\frac{2x}{x+3} = 5 + \frac{6x}{x+3}$

38. $-8 + \frac{64}{x-8} = \frac{x^2}{x-8}$

39. $\frac{9}{x^2-9} = \frac{3}{6(x-3)}$

40. Communicate Precisely Explain how to check if a solution to a rational equation is an extraneous solution.

41. Reason Diego and Stacy can paint a doghouse in 5 hours when working together. Diego works twice as fast as Stacy. Let x be the number of hours it would take Diego to paint the doghouse and y be the number of hours it would take Stacy to paint the doghouse. How long would it take Stacy to paint the doghouse if she was working alone? How long would it take Diego to paint the doghouse if he was working alone?