## UNDERSTAND

13. Model With Mathematics In the expression $P V^{\frac{4}{3}}$, $P$ represents the pressure and $V$ represents the volume of a sample of a gas. Evaluate the expression for $P=7$ and $V=8$.
14. Reason Describe the possible values of $k$ such that $\sqrt{32}+\sqrt{k}$ can be rewritten as a single term.
15. Error Analysis Explain why the following work is incorrect. Find the correct answer.

$$
\begin{aligned}
5\left(4-5^{\frac{1}{2}}\right) & =5(4)-5\left(5^{\frac{1}{2}}\right) \\
& =20-25^{\frac{1}{2}} \\
& =15
\end{aligned}
$$

16. Communicate Precisely Discuss the advantages and disadvantages of first rewriting $\sqrt{27}+\sqrt{48}+\sqrt{147}$ in order to estimate its decimal value.
17. Higher Order Thinking Write $\sqrt{\frac{4}{5}}$ in two different ways, one where the numerator is simplified and another where the denominator is rationalized.
18. Construct Arguments Justify each step used in simplifying the expression below.

$$
\begin{aligned}
\left(\frac{a^{2}}{\frac{3}{4}}\right)^{\frac{1}{5}} & =\left(a^{2-\frac{3}{4}}\right)^{\frac{1}{5}} \\
& =\left(a^{\frac{5}{4}}\right)^{\frac{1}{5}} \\
& =a^{\frac{1}{4}} \\
& =\sqrt[4]{a}
\end{aligned}
$$

## PRACTICE

What is the reduced radical form of each
expression? SEE EXAMPLE 1
19. $\left(3 x^{\frac{1}{2}}\right)\left(4 x^{\frac{2}{3}}\right)$
20. $2 b^{\frac{1}{2}\left(3 b^{\frac{1}{2}} c^{\frac{1}{3}}\right)^{2}}$
21. $\left(x^{\frac{1}{2}} \cdot x^{\frac{5}{12}}\right)^{4} \div x^{\frac{2}{3}}$
22. $\left(\frac{16 c^{14}}{81 d^{18}}\right)^{\frac{1}{2}}$

What is the reduced radical form of each expression? SEE EXAMPLE 2
23. $\sqrt[3]{250 y^{2} z^{4}}$
24. $\sqrt[4]{256 v^{7} w^{12}}$
25. $\sqrt{\frac{48 x^{3}}{3 x y^{2}}}$
26. $\sqrt{\frac{56 x^{5} y^{5}}{7 x y}}$
27. $\sqrt[3]{216 m}$
28. $\sqrt[3]{\frac{250 f^{7} g^{3}}{2 f^{2} g}}$

What is the reduced radical form of each expression? SEE EXAMPLE 3
29. $\sqrt{x^{5} y^{5}} \cdot 3 \sqrt{2 x^{7} y^{6}}$
30. $\sqrt[3]{\frac{18 n^{2}}{24 n}}$
31. $\sqrt[3]{3 x^{2}} \cdot \sqrt[3]{x^{2}} \cdot \sqrt[3]{9 x^{3}}$
32. $\sqrt{\frac{162 a}{6 a^{3}}}$
33. $\sqrt[5]{2 p q^{6}} \cdot 2 \sqrt{2 p^{3} q}$
34. $\sqrt[3]{\frac{x^{2}}{9 y}}$
35. $\sqrt[3]{6} \cdot \sqrt[3]{16}$
36. $\sqrt[4]{\frac{2}{5 x}}$

What is the reduced radical form of each expression? SEE EXAMPLE 4
37. $4 \sqrt[3]{81}-2 \sqrt[3]{72}-\sqrt[3]{24}$
38. $6 \sqrt{45 y^{2}}-4 \sqrt{20 y^{2}}$
39. $3 \sqrt{12}-\sqrt{54}+7 \sqrt{75}$
40. $\sqrt{32 h}+4 \sqrt{98 h}-3 \sqrt{50 h}$

Multiply. See example 5
41. $(3 \sqrt{p}-\sqrt{5})(\sqrt{p}+5 \sqrt{5})$
42. $(4 m-\sqrt{3})(4 m-\sqrt{3})$
43. $(3 \sqrt{2}+8)(3 \sqrt{2}-8)$
44. $\sqrt[3]{3}(5 \sqrt[3]{9}-4)$

What is the reduced radical form of each expression? SEE EXAMPLE 6
45. $\frac{4}{1-\sqrt{3}}$
46. $\frac{20}{3+\sqrt{2}}$
47. $\frac{3+\sqrt{8}}{2-2 \sqrt{8}}$
48. $\frac{-2 x}{3+\sqrt{x}}$

## APPLY

49. Model With Mathematics A triangular swimming area is marked off by a rope.
a. If a woman swims around the perimeter of the swimming area, how far will she swim?
b. What is the area of the roped off section?

50. Use Structure The interest rate $r$ required to increase your investment $p$ to the amount a in $m$ months is found by $r=\left(\frac{a}{p}\right)^{\frac{1}{m}}-1$. What interest rate would be required to increase your investment of $\$ 3,600$ to $\$ 6,400$ over 7 months? Round your answer to the nearest tenth of a percent.
51. Use Structure The length of a rectangle is $(2+\sqrt{5}) y$. The width is $(4+3 \sqrt{5}) z$. What is the area of the rectangle?

52. Model With Mathematics A rectangular boardroom table is $\sqrt{440} \mathrm{ft}$ by $\sqrt{20} \mathrm{ft}$. Find its area.

## ASSESSMENT PRACTICE

53. Aaron is rewriting $\frac{1+\sqrt{3}}{5-\sqrt{3}}$ into reduced radical form. Determine if Aaron would have written the steps below to show his work. Select Yes or No.

|  | Yes | No |
| :--- | :--- | :--- |
| $\frac{6+4 \sqrt{3}-3}{25+9}$ | $\square$ | $\square$ |
| $\frac{5+\sqrt{3}+5 \sqrt{3}+\sqrt{9}}{25+5 \sqrt{3}-5 \sqrt{3}-\sqrt{9}}$ | $\square$ | $\square$ |
| $\frac{4+3 \sqrt{3}}{11}$ | $\square$ | $\square$ |
| $\frac{8+6 \sqrt{3}}{28}$ | $\square$ | $\square$ |
| $\frac{5+6 \sqrt{3}+3}{25-3}$ | $\square$ | $\square$ |

54. SAT/ACT Which expression cannot be rewritten as -10 ?
(A) $\sqrt{25} \cdot \sqrt[3]{-8}$
(B) $\sqrt[3]{-125} \cdot \sqrt[4]{16}$
(C) $-\sqrt[3]{1,000}$
(D) $-\sqrt{25} \cdot \sqrt[5]{-32}$
(E) $\sqrt{4} \cdot-\sqrt[3]{125}$
55. Performance Task The volume of a sphere of radius $r$ is $V=\frac{4}{3} \pi r^{3}$.
Part A Use the formula to find $r$ in terms of $V$. Rationalize the denominator.

Part B A snowman is made using three spherical snowballs. The top snowball for the head has a volume of $500 \mathrm{in}^{3}$. What is the diameter of the top snowball?


Part C The volumes of the other two snowballs are $750 \mathrm{in.}^{3}$ and $1,000 \mathrm{in}^{3}$. How tall is the snowman?

