Topic 5 Review

Monday, September 7, 2020 8:51 AM

TOPIC 5

Topic Review

TOPIC ESSENTIAL QUESTION

1. How are rational exponents and radical equations used to solve real-world problems?

Vocabulary Review

Choose the correct term to complete each sentence.

2. In the expression $\sqrt[n]{c}$, n is the Index.
3. In the expression $\sqrt[n]{c}$, c is the radicand.
4. Radicals with the same index and the same radicand are like radicals.
5. A(n) radical function is a function defined by a radical expression.
6. A(n) nth root of a number c is x, such that $x^n = c$.
7. A(n) extraneous solution is a potential solution that must be rejected because it does not satisfy the original equation.
8. When all nth roots of perfect nth powers have been simplified and no radicals remain in the denominator, an expression is in reduced radical form.
9. A(n) Composite function results from the application of one function to the output of another function.

- composite function
- extraneous solution
- index
- inverse function
- like radicals
- nth root
- radical function
- radicand
- reduced radical form

Concepts & Skills Review

LESSON 5-1 nth Roots, Radicals, and Rational Exponents

Quick Review

An **nth root** of a number c is x, such that $x^n = c$. The nth root of c can be represented as $\sqrt[n]{c}$, where n is the **index** and c is the **radicand**.

Example

Solve the equation $2x^4 = 162$.

$2x^4 = 162$ Write the original equation.

$x^4 = 81$ Divide both sides by 2.

$(x^4)^{\frac{1}{4}} = (81)^{\frac{1}{4}}$ Raise both sides to the reciprocal of the exponent of x.

$x = 3$ Use the Power of a Power Property.

Practice & Problem Solving

What is the value of each expression? Round to the nearest hundredth, if necessary.

10. $\sqrt[4]{16^2}$ $2\cdot2\cdot2\cdot2$ $2^2 \to 4$

11. $-\sqrt[3]{25^6}$ $5\,5$ $-\sqrt[3]{5^{12}} \to -5^4 \to -625$

Simplify each expression.

odd

12. $\sqrt[3]{27x^{12}}$ $3\,3\,3$ $x\,x\,x\,x$ $\to 3x^4$

even

13. $\sqrt[4]{16a^{24}b^8}$ $2\,2\,2\,2$ $\to 2a^6b^2$

Solve each equation.

14. $750 = 6y^3$ 6 6

15. $1{,}280 = 5z^4$ 5 5

256, 1 16, 4 4 4 4

14) $125 = y^3$, $\sqrt[3]{125}$ $\sqrt[3]{y^3}$, $5\,5\,5$, $5 = y$

15) $256 = z^4$, $\pm\sqrt[4]{256}$ $\sqrt[4]{z^4}$, $\pm 4 = z$

16. **Communicate Precisely** Describe the relationship between a rational exponent and a root of a number x.

17. **Make Sense and Persevere** The function $d(t) = 9.8t^2$ represents how far an object falls, in meters, in t seconds. How long would it take a rock to fall from a height of 300 m? Round to the nearest hundredth of a second.

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18) $\sqrt{x^6x^8y^4y^6}$, $\sqrt{x^{14}y^{10}}$, x^7y^5

19) $\sqrt[3]{81m^3}$, $3\,3\,3\,3$, $3m\sqrt[3]{3}$

20) $\sqrt[3]{125x^9}$, $5\,5\,5$, $5x^3$

21) $\sqrt{49a^6}$ even $7|a^3|$

22) $n + 3\sqrt{7n} - \sqrt{7n} - 3\cdot7$, $n + 2\sqrt{7n} - 21$

23) $81x^2 + 9x\sqrt{2} + 9x\sqrt{2} + 2$, $81x^2 + 18x\sqrt{2} + 2$

24) $25\cdot3 - 30\sqrt{3} + 30\sqrt{3} - 36$, 75, 39

25) $6\sqrt[3]{4\cdot2} - 1\sqrt[3]{4}$, $\to 6\sqrt[3]{8}$, $\to 6\cdot2 - \sqrt[3]{4}$, $\to 12 - \sqrt[3]{4}$

26) $\frac{6}{1+\sqrt{2}}\frac{(1-\sqrt{2})}{(1-\sqrt{2})}$ $\to \frac{6-6\sqrt{2}}{1-2} \to \frac{6-6\sqrt{2}}{-1}$ $\to -6+6\sqrt{2}$

27) $\frac{5}{2-\sqrt{5}}\frac{(2+\sqrt{5})}{(2+\sqrt{5})}$ $\to \frac{10+5\sqrt{5}}{4-5} \to \frac{10+5\sqrt{5}}{-1}$ $\to -10-5\sqrt{5}$

28) $\frac{4+\sqrt{6}}{3-3\sqrt{6}}\frac{(3+3\sqrt{6})}{(3+3\sqrt{6})}$, $12 + 12\sqrt{6} + 3\sqrt{6} + 3\cdot6$, 18

29) $\frac{-9x}{\sqrt{x}}\frac{\sqrt{x}}{\sqrt{x}}$ $\to \frac{-9x\sqrt{x}}{x}$

LESSON 5-2 Properties of Exponents and Radicals

Quick Review

To simplify radical expressions, look for factors that are perfect nth power factors.

The Product Property of Radicals and Quotient Property of Radicals can also be used to rewrite radical expressions.

Product Property of Radicals $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

Quotient Property of Radicals $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Example

What is $\sqrt[4]{64} \cdot \sqrt[4]{2}$ in reduced radical form?

Practice & Problem Solving

What is the reduced radical form of each expression?

18. $\sqrt{x^6y^4} \cdot \sqrt{x^8y^6}$

19. $\sqrt[3]{\frac{243m^4}{3m}}$ 81, m^3

20. $\sqrt[3]{5x^4} \cdot \sqrt[3]{x^2} \cdot \sqrt[3]{25x^3}$

21. $\sqrt{\frac{98a^{10}}{2a^4}}$ 49, 6

Multiply. $\sqrt[3]{5\cdot25x^4x^2x^3}$

22. $(\sqrt{n} - \sqrt{7})(\sqrt{n} + 3\sqrt{7})$

23. $(9x + \sqrt{2})(9x + \sqrt{2})$

24. $(5\sqrt{3} + 6)(5\sqrt{3} - 6)$

25. $\sqrt[3]{4}\,(6\sqrt[3]{2} - 1)$

conjugate $a + \sqrt{b}$

How can you rewrite each expression so there ...

Quotient Property of Radicals $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Example

What is $\sqrt[4]{64} \cdot \sqrt[4]{2}$ in reduced radical form?

$\sqrt[4]{64} \cdot \sqrt[4]{2}$	Write the original expression.
$\sqrt[4]{64 \cdot 2}$	Use the Product Property of Radicals.
$\sqrt[4]{128}$	Multiply.
$\sqrt[4]{16} \cdot \sqrt[4]{8}$	Rewrite using the Product Property of Radicals.
$2\sqrt[4]{8}$	Simplify.

conjugates
$a+\sqrt{b}$ &
$a-\sqrt{b}$

22. $(\sqrt{n} - \sqrt{7})(\sqrt{n} + 3\sqrt{7})$
23. $(9x + \sqrt{2})(9x + \sqrt{2})$
24. $(5\sqrt{3} + 6)(5\sqrt{3} - 6)$
25. $\sqrt[3]{4}\,(6\sqrt[3]{2} - 1)$

How can you rewrite each expression so there are no radicals in the denominator?

rationalize denominator

FOIL

26. $\frac{6}{1 + \sqrt{2}}$
27. $\frac{5}{2 - \sqrt{5}}$
28. $\frac{4 + \sqrt{6}}{3 - 3\sqrt{6}}$
29. $\frac{-9x}{\sqrt{x}}$

30. **Error Analysis** Describe and correct the error made in rewriting the radical expression.

$5\sqrt{18} - \sqrt{27} = 7\sqrt{2}$

31. **Reason** A rectangular wall is $\sqrt{240}$ ft by $\sqrt{50}$ ft. You need to paint the wall twice to cover the area with two coats of paint. If each can of paint can cover 60 square feet, how many cans of paint will you need?

28) $\frac{(4+\sqrt{6})}{(3-3\sqrt{6})} \cdot \frac{(3+3\sqrt{6})}{(3+3\sqrt{6})}$

$\frac{12 + 12\sqrt{6} + 3\sqrt{6} + \underbrace{3\cdot 6}_{18}}{9 - 9\cdot 6}$

54

$\frac{30 + 15\sqrt{6}}{-45}$

$\rightarrow \frac{2+\sqrt{6}}{-3}$

29) $\frac{-9x}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}}$

$\rightarrow \frac{-9\cancel{x}\sqrt{x}}{\cancel{x}}$

$\rightarrow -9\sqrt{x}$

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LESSON 5-3 Graphing Radical Functions

Quick Review

A **radical function** is a function defined by a radical expression. To determine transformations of a radical function, write the radical function in the form $h(x) = a\sqrt[n]{x - h} + k$ and compare it to the parent function.

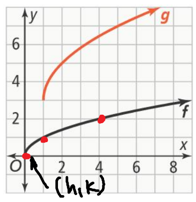
$a\sqrt[n]{x-h}+k$

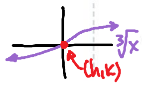
a: vert stretch/shrink/refl; h: shift L/R; k: shift U/D

Example

Graph $g(x) = 2\sqrt{x - 1} + 3$.

$g(x) = 2\sqrt{x - 1} + 3$ is a vertical stretch by a factor of 2, a horizontal shift 1 unit to the right, and a vertical shift 3 units up from the parent function $f(x) = \sqrt{x}$.





Practice & Problem Solving

Graph the following functions. What are the domain and range? Is the function increasing or decreasing?

32. $f(x) = \sqrt{x} - 1$ — k: −1 down
33. $f(x) = \sqrt[3]{x} + 2$ — k: 2 up
34. $f(x) = \frac{1}{2}\sqrt{x + 1}$ — a: ½ vert shrink; h: −1 left
35. $f(x) = 2\sqrt[3]{x} - 1$ — a: vert stretch; k: −1 down
36. $f(x) = \sqrt[3]{x - 3}$ — h: 3 right
37. $f(x) = \sqrt{x + 4} - 2$ — h: −4 left; k: −2 down

38. **Communicate Precisely** Explain how to rewrite the function $g(x) = \sqrt[3]{8x - 24} + 1$ to identify the transformations from the parent graph $f(x) = \sqrt[3]{x}$.

39. **Reason** The speed s, in miles per hour, of a car when it starts to skid can be estimated using the formula $s = \sqrt{30 \cdot 0.5d}$, where d is the length of the skid marks, in feet. Graph the function. If a car's skid marks measure 40 ft in a zone where the speed limit is 25 mph, was the car speeding? Explain.

32) d: $x \ge 0$, r: $y \ge -1$, increasing
33) d: ℝ, r: ℝ, increasing
34) d: $x \ge -1$, r: $y \ge 0$, increasing
35) d: ℝ, r: ℝ, increasing
36) d: ℝ, r: ℝ, increasing
37) d: $x \ge -4$, r: $y \ge -2$, increasing

38) $\sqrt[3]{8x-24}+1$
$\sqrt[3]{8(x-3)}+1$
$2\sqrt[3]{x-3}+1$
a: 2 vert stretch; h: 3 right; k: 1 up

LESSON 5-4 Solving Radical Equations

• Isolate radical.
• Solve for variable

Quick Review

To solve a radical equation, isolate the radical. Raise both sides of the equation to the appropriate power to eliminate the radical and solve for x. Then check for extraneous solutions. If the equation includes more than one radical, eliminate one radical at a time using a similar process.

Example

Solve the radical equation $\sqrt{6 - x} = x$.

$\sqrt{6 - x} = x$	Write the original equation.
$(\sqrt{6 - x})^2 = (x)^2$	Square both sides.
$6 - x = x^2$	Simplify.
$0 = x^2 + x - 6$	Write in standard form.
$0 = (x + 3)(x - 2)$	Factor.
$x = -3$ or $x = 2$	Use the Zero-Product Property.

Check the solutions to see if they both make the original equation true.

Practice & Problem Solving

Solve each radical equation. Check for extraneous solutions.

40. $\sqrt[3]{x} - 2 = 7$ (+2 +2)
41. $\sqrt{2x} = 12$
42. $\sqrt{25 + x} + 5 = 9$ (−5 −5)
43. $13 - \sqrt[4]{x} = 10$ (−13 −13)
44. $\sqrt{5x + 1} + 1 = x$ (−1 −1)
45. $\sqrt{6x - 20} - x = -6$ (+x +x)

46. **Construct Arguments** Give an example of a radical equation that has no real solutions. Explain your reasoning.

47. **Make Sense and Persevere** The formula $d = \frac{\sqrt{15w}}{3.14}$ gives the diameter d, in inches, of a rope needed to lift a weight of w, in tons. How much weight can be lifted with a rope that has a diameter of 4 in?

40) $\sqrt[3]{x} = 9$
$(\sqrt[3]{x})^3 = (9)^3$
$x = 729$

41) $2x = 144$
$x = 72$

42) $\sqrt{25+x} = 4$
$(\ )^2 = (4)^2$
$25 + x = 16$
$-25 \quad -25$
$x = -9$

43) $-\sqrt[4]{x} = -3$
$\sqrt[4]{x} = 3$
$(\sqrt[4]{x})^4 = (3)^4$
$x = 81$

44) $\sqrt{5x+1} = x - 1$
$(\sqrt{5x+1})^2 = (x-1)^2$
$5x + 1 = (x-1)(x-1)$
$5x + 1 = x^2 - x - x + 1$
$-5x - 1 \quad -5x - 1$
$0 = x^2 - 7x$
$0 = x(x-7)$
$\cancel{x = 0}$ $x = 7$ ✓
extraneous

45) $\sqrt{6x-20} = x - 6$
$6x - 20 = (x-6)(x-6)$
$6x - 20 = x^2 - 6x - 6x + 36$
$-6x + 20 \quad -6x + 20$
$0 = x^2 - 18x + 56$
$-1, -56$; $-2, -28$; $-4, -14$
$0 = (x-4)(x-14)$
$\cancel{x = 4}$ $x = 14$ ✓
extraneous

LESSON 5-5 Function Operations

Quick Review

You can add, subtract, multiply, or divide functions. When adding and subtracting functions, the domain is the intersection of the domains of the two functions. When multiplying and dividing functions, the domain is the set of all real numbers for which both original functions and the new function are defined. You can also compose functions, by using one function as the input for another function. These are called **composite functions**.

$f \circ g$

turkey stuffing

Example

Let $f(x) = 5x$ and $g(x) = 3x - 1$. What is the rule for the composition $f \circ g$?

$f \circ g = f(g(x))$ ······ Apply the definition.

$= f(3x - 1)$ ······ Apply the rule for g.

$= 5(3x - 1)$ ······ Apply the rule for f.

$= 15x - 5$ ······ Distribute.

Practice & Problem Solving

Let $f(x) = -x + 6$ and $g(x) = 5x$. Identify the rule for the following functions.

48. $f + g$

49. $f - g$

50. $g(f(2))$

51. $f(g(-1))$

52. **Reason** For the functions f and g, what is the domain of $f \circ g$? $\frac{f}{g}$? $\frac{g}{f}$?

53. **Make Sense and Persevere** A test has a bonus problem. If you get the bonus problem correct, you will receive 2 bonus points and your test score will increase by 3% of your score. Let $f(x) = x + 2$ and $g(x) = 1.03x$, where x is the test score without the bonus problem. Find $g(f(78))$. What does $g(f(78))$ represent?

48) $-x+6+5x \rightarrow 4x+6$

49) $-x+6-5x \rightarrow -6x+6$

50) $f(2) = -(2)+6$
$= -2+6$
$= 4$
$g(f(2)) \rightarrow g(4)$
$\rightarrow 5(4)$
$\rightarrow 20$

51) $g(-1) = 5(-1)$
$= -5$
$f(g(-1))$
$f(-5) = -(-5)+6$
$= 5+6$
$= 11$

LESSON 5-6 Inverse Relations and Functions

• Swap x & y
• Isolate y

Quick Review

An **inverse relation** is formed when the roles of the independent and dependent variables are reversed. If an inverse relation of a function, f, is itself a function, it is called the **inverse function** of f, which is written $f^{-1}(x)$.

$f^{-1}(x) \neq \frac{1}{f(x)}$

Example

What is the inverse of the relation represented in the table?

x	y
−2	0
−1	6
0	5
1	3
3	−1

Switch the values of x and y. Then reorder the ordered pairs.

x	y
−1	3
0	−2
3	1
5	0
6	−1

Practice & Problem Solving

Find an equation of the inverse function.

54. $f(x) = -4x^2 + 3$

55. $f(x) = \sqrt{x - 4}$

56. $f(x) = 9x + 5$

57. $f(x) = \sqrt{x + 7} - 1$

58. **Error Analysis** Jamie said the inverse of $f(x) = \sqrt{x - 9}$ is $f^{-1}(x) = (x + 9)^2$. Is Jamie correct? Explain.

59. **Make Sense and Persevere** An electrician charges \$50 for a house visit plus \$40 per hour. Write a function for the cost C of an electrician charging for h hours. Find the inverse of the function. If the bill is \$150, how long did the electrician work?

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54) $x = -4y^2 + 3$
$-3 \quad -3$
$x - 3 = -4y^2$
$\div -4 \quad \div -4$
$\frac{x-3}{-4} = y^2$
$\pm\sqrt{} \quad \sqrt{y^2}$
$\boxed{\pm\sqrt{\frac{x-3}{-4}} = y}$

55) $x = \sqrt{y-4}$
$(x)^2 \quad (\sqrt{y-4})^2$
$x^2 = y - 4$
$+4 \quad +4$
$\boxed{x^2 + 4 = y}$ $f^{-1}(x)$

56) $x = 9y + 5$
$-5 \quad -5$
$x - 5 = 9y$
$\div 9 \quad \div 9$
$\boxed{\frac{x-5}{9} = y}$ $f^{-1}(x)$

57) $x = \sqrt{y+7} - 1$
$+1 \quad +1$
$x + 1 = \sqrt{y+7}$
$()^2 \quad ()^2$
$(x+1)^2 = y + 7$
$-7 \quad -7$
$\boxed{(x+1)^2 - 7 = y}$ $f^{-1}(x)$