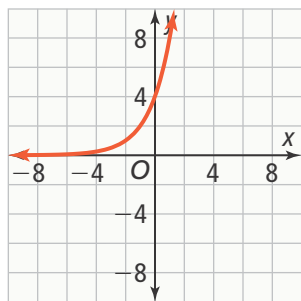# ✎ PRACTICE & PROBLEM SOLVING

UNDERSTAND

9. Use Structure What value of a completes the equation $y = a \cdot 2^x$ for the exponential growth function shown below?

10. Make Sense and Persevere Cindy found a collection of baseball cards in her attic worth $8,000. The collection is estimated to increase in value by 1.5% per year. Write an exponential growth function and find the value of the collection after 7 years.

11. Error Analysis Describe and correct the error a student made in identifying the growth or decay factor for the function $y = 2.55(0.7)^x$.

> Step 1 The base of the function is 0.7, so it represents exponential decay.
> Step 2 The function in the form $y = a(1 - r)^x$ is $y = 2.55(1 - 0.7)^x$.
> Step 3 The decay factor is 0.3.

✗

12. Reason In 2000, the population of St. Louis was 346,904, and it decreased to 319,257 in 2010. If this population decrease were modeled by an exponential decay function, what value would represent the y-intercept? Explain your reasoning.

13. Mathematical Connections Describe how the graph of $g(x) = 6 \cdot 2^{x+1} - 4$ compares to the graph of $f(x) = 6 \cdot 2^x$.

PRACTICE

Identify the domain, range, intercept, and asymptote of each exponential function. Then describe the end behavior. **SEE EXAMPLE 1**

14. $f(x) = 5 \cdot 3^x$

15. $f(x) = 0.75\left(\frac{2}{3}\right)^x$

16. $f(x) = 4\left(\frac{1}{2}\right)^x$

17. $f(x) = 7 \cdot 2^x$

Determine whether each function represents exponential growth or decay. Write the base in terms of the rate of growth or decay, identify r, and interpret the rate of growth or decay.
SEE EXAMPLES 3 AND 4

18. $y = 100 \cdot 2.5^x$

19. $f(x) = 10,200\left(\frac{3}{5}\right)^x$

20. $f(x) = 12,000\left(\frac{7}{10}\right)^x$

21. $y = 450 \cdot 2^x$

22. The function $f(x)$, shown in the graph, represents an exponential growth function. Compare the average rate of change of $f(x)$ to the average rate of change of the exponential growth function $g(x) = 25\,(1.4)^x$. Use the interval $[0, 4]$. **SEE EXAMPLE 5**
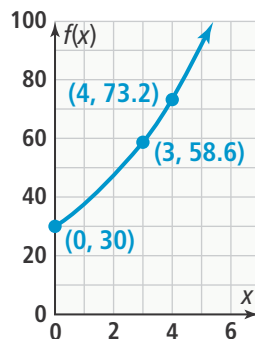
23. Write a function $g(x)$ that represents the exponential function $f(x) = 2^x$ after a vertical stretch of 6 and a reflection across the x-axis. Graph both functions. **SEE EXAMPLE 2**

24. The population of Medway, Ohio, was 4,007 in 2000. It is expected to decrease by about 0.36% per year. Write an exponential decay function and use it to approximate the population in 2020. **SEE EXAMPLE 4**

APPLY

25. Model With Mathematics A colony of bacteria starts with 50 organisms and quadruples each day. Write an exponential function, $P(t)$, that represents the population of the bacteria after t days. Then find the number of bacteria that will be in the colony after 5 days.

DAY 0: 50   DAY 5: ?

26. Higher Order Thinking The number of teams y remaining in a single elimination tournament can be found using the exponential function $y = 128\left(\frac{1}{2}\right)^x$, where x is the number of rounds played in the tournament.

a. Determine whether the function represents exponential growth or decay. Explain.

b. What does 128 represent in the function?

c. What percent of the teams are eliminated after each round? Explain how you know.

d. Graph the function. What is a reasonable domain and range for the function? Explain.

27. Construct Arguments The function shown in the graph represents the number of lions in a region after x years, where the rate of decay is 20%. The number of zebras in that same region after x years can be modeled by the function $f(x) = 300(0.95)^x$. A representative for a conservationist group claims there will be fewer lions than zebras within 2 years. Is the representative correct? Justify your answer.

Lion Population

ASSESSMENT PRACTICE

28. The exponential function $g(x) = 3^{x-1} + 6$ is a transformation of the function $f(x) = 3^x$. Does each statement accurately describe how the graph of $g(x)$ compares to the graph of $f(x)$? Select yes or no.

	Yes	No
a. $g(x)$ is translated 6 units up.	❑	❑
b. $g(x)$ is translated 6 units down.	❑	❑
c. $g(x)$ is translated 6 units to the right.	❑	❑
d. $g(x)$ is translated 1 unit to the right.	❑	❑
e. $g(x)$ is translated 1 unit to the left.	❑	❑
f. The horizontal asymptote shifts 1 unit down.	❑	❑

29. SAT/ACT Which of the functions defined below could be the one shown in this graph?
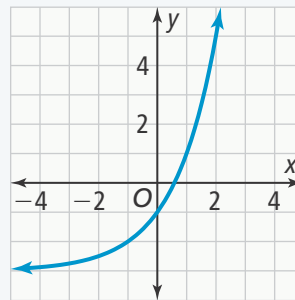
Ⓐ $f(x) = 4(2)^{x-1} + 3$ Ⓒ $f(x) = 4(2)^{x-1} - 3$

Ⓑ $f(x) = 4(2)^{x+1} + 3$ Ⓓ $f(x) = 4(2)^{x+1} - 3$

30. Performance Task A radioactive isotope of the element osmium Os-182 has a half-life of 21.5 hours. This means that if there are 100 grams of Os-182 in a sample, after 21.5 hours there will only be 50 grams of that isotope remaining.

Part A Write an exponential decay function to model the amount of Os-182 in a sample over time. Use A_0 for the initial amount and A for the amount after time t in hours.

Part B Use your model to predict how long it would take a sample containing 500 g of Os-182 to decay to the point where it contained only 5 g of Os-182.