Topic 6 Review

Monday, September 7, 2020 8:52 AM

TOPIC 6

Topic Review

TOPIC ESSENTIAL QUESTION

1. How do you use exponential and logarithmic functions to model situations and solve problems?

Vocabulary Review

Choose the correct term to complete each sentence.

2. A(n) natural logarithm has base e.
3. A(n) exponential function has the form $f(x) = a \cdot b^x$.
4. In an exponential function, when $0 < b < 1$, b is a(n) decay factor.
5. The Change of base allows logarithms with a base other than 10 or e to be evaluated.
6. A(n) Common logarithm has base 10.
7. The inverse of an exponential function is a(n) logarithmic function.

- ~~decay factor~~
- ~~exponential function~~
- ~~logarithmic function~~
- growth factor
- ~~common logarithm~~
- ~~natural logarithm~~
- ~~Change of Base Formula~~

Concepts & Skills Review

LESSON 6-1 Key Features of Exponential Functions

Quick Review

An **exponential function** has the form $f(x) = a \cdot b^x$. When $a > 0$ and $b > 1$, the function is an **exponential growth function**. When $a > 0$ and $0 < b < 1$ the function is an **exponential decay function**.

Example

Paul invests $4,000 in an account that pays 2.5% interest annually. How much money will be in the account after 5 years?

Write and use the exponential growth function model.

$A(t) = a(1 + r)^n$

$A(5) = 4{,}000(1 + 0.025)^5$

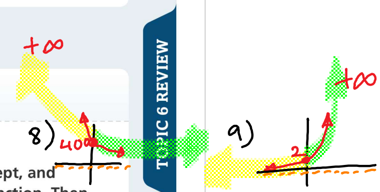
$A(5) = 4{,}000(1.025)^5$

$A(5) = 4{,}525.63$

There will be about $4,525.63 in Paul's account after 5 years.

Practice & Problem Solving

Identify the domain, range, intercept, and asymptote of each exponential function. Then describe the end behavior.



8. $f(x) = 400 \cdot \left(\frac{1}{2}\right)^x$ ← b: decay

d: ℝ / r: y>0 / set x=0 y-int: 400 / horiz asymp: y=0 / As $x \to -\infty$, $y \to +\infty$; As $x \to +\infty$, $y \to 0$

9. $f(x) = 2 \cdot (3)^x$ ← b: growth

d: ℝ / r: y>0 / set x=0 y-int: 2 / horiz asymp: y=0 / As $x \to -\infty$, $y \to 0$; As $x \to +\infty$, $y \to +\infty$

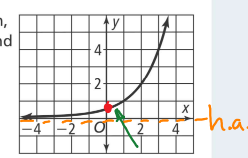
10. **Reason** Seth invests $1,400 at 1.8% annual interest for 6 years. How much will Seth have at the end of the sixth year?

10) $A(t) = a(1+r)^n$
$= 1400(1+.018)^6$
$= \$1558.17$

11. **Model With Mathematics** Bailey buys a car for $25,000. The car depreciates in value 18% per year. How much will the car be worth after 3 years?

11) $A(t) = a(1-r)^n$
$= 25000(1-.18)^3$
$= \$13784.20$

12. Identify the domain, range, intercept, and asymptote.



d: ℝ
r: y>0
y-int: $\frac{1}{2}$
asymp: y=0

LESSON 6-2 Exponential Models

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Final, principal, 100%, rate, time, # times interest is compounded per year

Quick Review

Compound Interest

Interest may be compounded over different time periods, such as quarterly, monthly, or daily. The formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$ is used to calculate the amount of money available after it has been invested for an amount of time. Interest may also be compounded continuously. The formula $A = Pe^{rt}$ is used to calculate the amount of money available in an account that is compounded continuously. The calculator can be used to find an exponential model for a set of data.

natural base: e: 2.718

Example

Jenny invests $2,500 in an account that pays 2.4% interest annually. The interest is compounded quarterly. How much will Jenny have in the account after 6 years?

Use the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$.

Practice & Problem Solving

Find the total amount of money in the account after the given amount of time.

13. Compounded quarterly, $P = \$12{,}000$, $r = 3.6\%$, $t = 4$ years (n=4)

13) $A = 12000\left(1 + \frac{.036}{4}\right)^{(4 \cdot 4)} = \$13849.69$

14. Compounded monthly, $P = \$5{,}000$, $r = 2.4\%$, $t = 8$ years (n=12)

14) $5000\left(1 + \frac{.024}{12}\right)^{(12 \cdot 8)} = \$6057.19$

15. Continuously compounded, $P = \$7{,}500$, $r = 1.6\%$, $t = 10$ years

$A = Pe^{rt}$

15) $7500e^{(.016 \cdot 10)} = \$8801.33$

Write an exponential model given two points.

16. (12, 256) and (13, 302)

16) $\cancel{f(x)} = a \cdot b^x$ (y); b: growth factor

$y = a \cdot \left(\frac{302}{256}\right)^x$

$302 = a\left(\frac{302}{256}\right)^{13}$

$\frac{302}{\left(\frac{302}{256}\right)^{13}} = \frac{a\left(\frac{302}{256}\right)^{13}}{\left(\frac{302}{256}\right)^{13}}$

$35.24 = a$

$\therefore f(x) = 35.24(1.18)^{13}$

17. (3, 54) and (4, 74)

17) $\cancel{f(x)} = a \cdot b^x$ (y)

$y = a \cdot \left(\frac{74}{54}\right)^x$

$74 = a\left(\frac{74}{54}\right)^4$

$\frac{74}{\left(\frac{74}{54}\right)^4} = \frac{a\left(\frac{74}{54}\right)^4}{\left(\frac{74}{54}\right)^4}$

$20.98 = a$

$\therefore f(x) = 20.98(1.37)^x$

18. **Model With Mathematics** Jason's parents invested some money for Jason's education when Jason was born. The table shows how the account has grown

Jenny invests \$2,500 in an account that pays 2.4% interest annually. The interest is compounded quarterly. How much will Jenny have in the account after 6 years?

Use the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$.

$A = 2{,}500\left(1 + \frac{0.024}{4}\right)^{4(6)}$ ······ Substitute for A, P, n, and r.

$A = 2{,}500(1.006)^{24}$ ······ Simplify.

$A = 2{,}885.97$ ······ Use a calculator.

Jenny will have about \$2,885.97.

17. (3, 54) and (4, 74) γ

18. **Model With Mathematics** Jason's parents invested some money for Jason's education when Jason was born. The table shows how the account has grown.

Number of Years	Amount (\$)
1	2,250
3	2,525
6	3,480
7	4,400
9	6,000
13	9,250

Predict how much will be in the account after 18 years.

$35.24 = a$

$\therefore \boxed{f(x) = 35.24(1.18)^{13}}$

$20.98 = a$

$\therefore \boxed{f(x) = 20.98(1.37)^{x}}$

LESSON 6-3 Logarithms

Quick Review

A logarithm is an exponent. Common logarithms have base 10 and natural logarithms have base e. Exponential expressions can be rewritten in logarithmic form, and logarithmic expressions can be converted to exponential form.

$5^3 = 125$ can be rewritten as $\log_5 125 = 3$.

$\log 100 = 2$ can be rewritten as $10^2 = 100$.

argument, power, base, base 10, 2.718

Example

Evaluate $\log_2 \frac{1}{8}$.

$\log_2 \frac{1}{8} = x$	Write an equation.
$2^x = \frac{1}{8}$	Rewrite the equation in exponential form.
$2^x = 2^{-3}$	Rewrite the equation with a common base.
$x = -3$	Since the two expressions have a common base, the exponents are equal.

Practice & Problem Solving

Use Structure If an equation is given in exponential form, write the logarithmic form. If an equation is given in logarithmic form, write the exponential form.

exp→log, exp→log

19. $4^3 = 64$ $\log_4 64 = 3$
20. $10^2 = 100$ $\log_{10} 100 = 2$

log→exp, log→exp

21. $\log_6 216 = 3$ $6^3 = 216$
22. $\ln 20 = x$ $\log_e 20 = x$ $e^x = 20$

Evaluate each logarithmic expression.

log → exp

23. $\log_8 \frac{1}{64} = x$
24. $\log_3 81 = x$

Use Appropriate Tools Evaluate each logarithmic expression using a calculator. Round answers to the nearest thousandth.

25. $\log 628$ 2.798
26. $\ln 0.55$ -0.598

Evaluate each logarithmic expression.

27. $\log_5 5^9 = x$
28. $7^{\log_7 49} = x$

log→ exp: $5^x = 5^9$, $\boxed{x = 9}$

power; exp → log: $\log_7 x = \log_7 49$, $\boxed{x = 49}$

→ same base..

23) $8^x = 64$, $\cancel{8^x = 8^2}$, $\cancel{x = 2}$ Incorrect

$8^x = \frac{1}{64}$

$8^x = \frac{1}{8^2}$

$\boxed{8^x = 8^{-2}}$

24) $3^x = 81$, $3^x = 3^4$, $\boxed{x = 4}$

LESSON 6-4 Logarithmic Functions

Quick Review

A logarithmic function is the inverse of an exponential function.

Example

• Swap x & y • Isolate y (log, exp)

Find the inverse of $f(x) = 10^{x-2}$. Identify any intercepts or asymptotes.

$y = 10^{x-2}$	Write in $y = f(x)$ form.
$x = 10^{y-2}$	Interchange x and y.
$y - 2 = \log x$	Write in log form.
$y = \log x + 2$	Solve for y.

The equation of the inverse is $f^{-1}(x) = \log x + 2$. It has an x-intercept at $x = \frac{1}{100}$ and a vertical asymptote at the y-axis.

Inverse notation: $f^{-1}(x) \neq \frac{1}{f(x)}$

Practice & Problem Solving

• Find inverse; graph; graph inverse

Look for Relationships Graph each function and identify the domain and range. List any intercepts or asymptotes. Describe the end behavior.

29. $f(x) = \log_4 x$
30. $f(x) = \ln x - 2$

Use Structure Find the equation of the inverse of each function.

31. $f(x) = 8^{x-2}$
32. $f(x) = \frac{5^{x-2}}{8}$

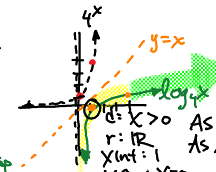
31. exp→log: $x = 8^{y-2}$, $\log_8 x = y - 2$, $(\log_8 x) + 2 = y = f^{-1}(x)$

32. exp→log: $8 \cdot x = 5^{y-2}$, $8x = 5^{y-2}$, $\log_5 8x = y - 2$, $(\log_5 8x) + 2 = y = f^{-1}(x)$

29) $x = \log_4 y$ (log→exp) $4^x = y$

x	y
0	1
1	4

swap



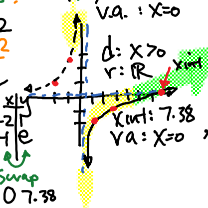
d: $x > 0$; r: $\mathbb{R}$; x-int: 1; V.A.: $x = 0$

As $x \to -\infty$, $y \to -\infty$; As $x \to +\infty$, $y \to +\infty$

30) $x = \ln y - 2$; $x + 2 = \log_e y$ (log→exp) $e^{x+2} = y$, h: -2

x	y
-2	1
-1	e

swap: 0, 7.38



d: $x > 0$; r: $\mathbb{R}$; x-int: 7.38; VA: $x = 0$

As $x \to -\infty$, $y \to -\infty$; As $x \to +\infty$, $y \to +\infty$

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LESSON 6-5 Properties of Logarithms

power prop of logs $\log m^r = r \cdot \log m$

Quick Review

Properties of logarithms can be used to either expand a single logarithmic expression into individual logarithms or condense several logarithmic expressions into a single logarithm.

The Change of Base Formula can be used to find logarithms of numbers with bases other than 10 or e.

Example

Use the properties of logarithms to expand the expression $\log_6 \frac{x^3y^5}{z}$.

$\log_6 \frac{x^3y^5}{z}$

$= \log_6 x^3y^5 - \log_6 z$ .... Quotient Property of Logarithms

$= \log_6 x^3 + \log_6 y^5 - \log_6 z$ .... Product Property of Logarithms

$= 3\log_6 x + 5\log_6 y - \log_6 z$ .... Power Property of Logarithms

Practice & Problem Solving

Use Structure Use the properties of logarithms to write each as a single logarithm. (condense) quotient product

33. $3\log r - 2\log s + \log t$ $\quad \log r^3 - \log s^2 + \log t$ / $\boxed{\log \frac{r^3 \cdot t}{s^2}}$

34. $2\ln 3 + 4\ln 2 - \ln 36$ $\quad \ln 3^2 + \ln 2^4 - \ln 36$ prod quot / $\ln \frac{3^2 \cdot 2^4}{36}$ / $\ln \frac{\cancel{9 \cdot 16}}{\cancel{36}} 4$ / $\boxed{\ln 4}$

Evaluate each logarithm. Change of Base

35. $\log_4 12 = x$ $\quad 4^x = 12$

36. $\log_7 70 = x$ $\quad 7^x = 70$

35) $\log_4 12 = \frac{\log_{10} 12}{\log_{10} 4} \approx \boxed{1.792}$

36) $\log_7 70 = \frac{\log 70}{\log 7} \approx \boxed{2.183}$ aka $\frac{\ln 70}{\ln 7}$

Make Sense and Persevere Use the Change of Base Formula to solve each equation for *x*. Give an exact solution written as a logarithm and an approximate solution rounded to the nearest thousandth.

37. $5^x = 200$ exp → log

$\log_5 200 = x$

$\frac{\log 200}{\log 5} = x$

$3.292 = x$

38. $7^x = 486$

$\log_7 486 = x$

$\frac{\log 486}{\log 7} = x$

$= x$

LESSON 6-6 Exponential and Logarithmic Equations

Quick Review

You can solve exponential equations by taking the logarithm of both sides. You can solve a logarithmic equation by combining the logarithmic terms into one logarithm and then converting to exponential form.

Example

Solve $7^{2x} = 10^{x+1}$.

$7^{2x} = 10^{x+1}$

$\log 7^{2x} = \log 10^{x+1}$ .... Take the common log of each side.

$2x \log 7 = (x + 1) \log 10$ .... Power Property of Logarithms

$2x \log 7 = x + 1$ .... Since $\log 10 = 1$

$2x \log 7 - x = 1$ .... Subtract *x* from each side.

$x(2 \log 7 - 1) = 1$ .... Factor out *x*.

$x = \frac{1}{2 \log 7 - 1}$ .... Divide each side by $2 \log 7 - 1$.

$x \approx 1.449$ .... Use a calculator.

Practice & Problem Solving

Find all solutions of the equation. Round answers to the nearest ten-thousandth.

39. $2^{5x+1} = 8^{x-1}$

40. $9^{2x+3} = 27^{x+2}$

41. $3^{x-2} = 5^{x-1}$

42. $7^{x+1} = 12^{x-1}$

log both sides

39) $2^{5x+1} = 8^{x-1}$

$2^{5x+1} = (2^3)^{x-1}$

$2^{5x+1} = 2^{3x-3}$ same base

$5x + 1 = 3x - 3$

$-3x - 1 \quad -3x - 1$

$2x = -4$

$\boxed{x = -2}$

40) $9^{2x+3} = 27^{x+2}$

$(3^2)^{2x+3} = (3^3)^{x+2}$

$3^{4x+6} = 3^{3x+6}$

$4x + 6 = 3x + 6$

$-3x - 6 \quad -3x - 6$

$\boxed{x = 0}$

41) $\log 3^{x-2} = \log 5^{x-1}$ power prop of logs

$(x-2)\log 3 = (x-1)\log 5$

$x\log 3 - 2\log 3 = x\log 5 - \log 5$

$-x\log 5 + 2\log 3 \quad -x\log 5 + 2\log 3$

$x(\log 3 - \log 5) = -\log 5 + 2\log 3$

$(\log 3 - \log 5) \quad (\log 3 - \log 5)$

$x = \boxed{-1.1507}$

42) $\log 7^{x+1} = \log 12^{x-1}$

$(x+1)\log 7 = (x-1)\log 12$

$x\log 7 + \log 7 = x\log 12 - \log 12$

$-x\log 12 - \log 7 \quad -x\log 12 - \log 7$

$x(\log 7 - \log 12) = -\log 12 - \log 7$

$(\log 7 - \log 12) \quad (\log 7 - \log 12)$

$\boxed{x = 8.220}$

Find all solutions of the equation. Round answers to the nearest ten thousandth. log → exp

43. $\log_5 (3x - 2)^4 = 8$

44. $\ln (x^2 - 32) = \ln (4x)$

45. $\log_6 (2x - 1) = 2 - \log_6 x$

46. **Model With Mathematics** Geri has \$1,500 to invest. He has a goal to have \$3,000 in this investment in 10 years. At what annual rate, compounded continuously, will Geri reach his goal? Round the answer to the nearest thousandth.

43) $5^8 = (3x-2)^4$

$\sqrt[4]{5^8} = \sqrt[4]{(3x-2)^4}$

$\sqrt[4]{5^8} = 3x - 2$

$+2 \quad +2$

$\sqrt[4]{5^8} + 2 = 3x$

$3 \quad 3$

$\boxed{9 = x}$

44) $x^2 - 32 = 4x$

$-4x \quad -4x$

$x^2 - 4x - 32 = 0$

$(x - 8)(x + 4) = 0$

$x - 8 = 0 \quad x + 4 = 0$

$x = 8 \quad x = \cancel{-4}$ extraneous

45) $\log_6 (2x-1) + \log_6 x = 2$ product prop

$\log_6 [(2x-1)x] = 2$ log → exp

$6^2 = 2x^2 - x$

$36 - 36 \quad -36$

$0 = 2x^2 - x - 36$

$0 = (2x - 9)(x + 4)$

$2x - 9 = 0 \quad x + 4 = 0$

$x = \frac{9}{2} \quad x = \cancel{-4}$ extraneous

	$2x$	-9
x	$2x^2$	$-9x$
4	$8x$	-36

$-72x^2$; $-9x$, $8x$; $-1x$

LESSON 6-7 Geometric Sequences and Series

Quick Review

A geometric sequence is defined by a common ratio between consecutive terms. It can be defined explicitly or recursively. A geometric series is the sum of the terms of a geometric sequence.

Example

A geometric sequence is defined by $a_n = \begin{cases} \frac{1}{9}, & n = 1 \\ 3a_{n-1}, & n > 1 \end{cases}$. What is the sum of the first 10 terms of this sequence?

$a_n = \frac{1}{9}(3)^{n-1}$ .... Write the explicit definition.

$a_{10} = \frac{1}{9}(3)^9 = 2187$ .... Find the 10^{th} term.

$S_{10} = \frac{\frac{1}{9}(1 - 3^{10})}{(1 - 3)} = 3280\frac{4}{9}$ .... Calculate the sum.

Practice & Problem Solving

Determine whether or not each sequence is geometric.

47. 2, 4, 6, 8, 10, 12, …

48. 2, 4, 8, 16, 32, 64, …

Convert between recursive and explicit forms.

49. $a_n = \begin{cases} \frac{1}{8}, & n = 1 \\ \frac{3}{2}a_{n-1}, & n > 1 \end{cases}$

50. $a_n = -2(5)^{n-1}$

Find the sum for each geometric series.

51. $\sum_{k=1}^{8} 3(2)^k$

52. $\sum_{k=1}^{9} 243\left(\frac{1}{3}\right)^k$

53. **Look for Relationships** Find the difference $\sum_{k=1}^{10} 5(2)^k - \sum_{k=1}^{10} 10(2)^k$. Explain how you found your answer.

54. **Make Sense and Persevere** The half-life of carbon-14 is 5,730 years. This is the amount

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$\sum_{k=1} 5(2)^{k} - \sum_{k=1} 10(2)^{k}$. Explain how you found your answer.

54. **Make Sense and Persevere** The half-life of carbon-14 is 5,730 years. This is the amount of time it takes for half of a sample to decay. From a sample of 24 grams of carbon 14, how long will it take until only 3 grams of the sample remains?

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